Rough Heston model overview

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Classical vs. Rough Heston Volatility Paths

Classical Heston Model

The classical Heston model assumes that the variance process V_t follows a Cox–Ingersoll–Ross (CIR) dynamics:

$$dV_t = \kappa(\theta - V_t)\,dt + \xi\sqrt{V_t}\,dW_t.$$

Key properties:

- Variance paths are smooth and mean-reverting.
- Short-term fluctuations are mild.
- Dynamics are Markovian.

Numerical example (smooth behavior). A representative classical Heston volatility path may evolve as:

$$V_t = 0.040,\ 0.045,\ 0.050,\ 0.055,\ 0.060,$$

showing gradual changes and no sharp jumps.

Rough Heston Model

The Rough Heston model modifies volatility via a fractional kernel:

$$V_t = V_0 + \int_0^t (t-s)^{H-\frac{1}{2}} \left[\kappa(\theta - V_s)\,ds + \xi\sqrt{V_s}\,dW_s\right],$$

where $0 < H < 0.5$ is the Hurst exponent.

Key properties:

- Volatility exhibits high irregularity with sharp, frequent spikes.
- Strong volatility clustering and long-memory effects.
- Dynamics are non-Markovian.
- Empirically consistent with realized volatility ($H \approx 0.1$).

Conceptual Example of Rough Heston

Rough volatility arises from a "memory kernel" that gives recent shocks large weight and very old shocks small but nonzero weight:

$$V_t \sim \sum_{j=0}^{t} (t-j)^{H-1/2} \cdot Shock_j.$$

For $H = 0.1$, the kernel behaves roughly like

$$(t-j)^{-0.4},$$

which decays *slowly*. Conceptually:

- A volatility shock today still influences volatility far into the future.
- Many overlapping, slowly-decaying effects create jagged, fractal variability.
- This produces bursty volatility clusters even without jumps.

A simplified illustration: If the market receives the following shocks

$$+1, \quad -2, \quad +3, \quad -1, \quad +2,$$

the rough kernel gives weights roughly

$$1.00,\ 0.76,\ 0.57,\ 0.44,\ 0.34.$$

Thus the effective impact becomes

$$(+1)1.00 + (-2)0.76 + (+3)0.57 + (-1)0.44 + (+2)0.34,$$

yielding a highly irregular combined effect. This is qualitatively unlike the smooth, nearly exponential decay in classical Heston.

A Simple (Discrete) Rough Heston Variance Example

We consider the discrete approximation:

$$V_{t+1} = V_0 + \sum_{j=0}^{t} K(t-j) \left[\kappa(\theta - V_j)\,\Delta t + \xi \sqrt{V_j}\,\Delta W_j \right], \qquad K(\tau) = \tau^{H-\frac{1}{2}}.$$

Use the parameters:

$$V_0 = 0.04, \qquad \kappa = 2.0, \qquad \theta = 0.05, \qquad \xi = 0.5, \qquad H = 0.1.$$

Step 1: computing V_1. With $\Delta W_0 = 0.1$ and $K(1) = 1$,

$$V_1 = 0.04 + 1\left[2(0.05 - 0.04) + 0.5\sqrt{0.04}(0.1)\right] = 0.07.$$

Step 2: computing V_2. Kernel values:

$$K(2) = 2^{-0.4} \approx 0.76, \qquad K(1) = 1.$$

With $\Delta W_1 = -0.05$:

$$V_2 \approx 0.04 + 0.76(0.03) - 0.005 = 0.0578.$$

Resulting rough path segment.

$$V_0 = 0.040, \qquad V_1 = 0.070, \qquad V_2 \approx 0.0578.$$

This already shows higher irregularity than the classical Heston example.

Comparison

- Classical Heston: smooth, gently mean-reverting paths.
- Rough Heston: jagged, fractal-like paths with long memory and clustering.
- Even short discrete examples show clear differences for $H \approx 0.1$.